Math 61
Midterm I
February 7, 2011

Name:____________________, ____________________
Please put your last name first and print clearly

Signature:______________________________

TA section you are attending:
(Tues or Thurs, name of TA):_______________

1.__________

2.__________

3.__________

4.__________

5.__________

Total__________

You must put all your answers in the spaces provided on the page of the problem . **Please do not use the spaces on this page.** You must show a method of solution to obtain credit for a problem. You need not simplify your answers and you can leave your answers in terms of C(n,r)=$\binom{n}{r}$ or P(n,r).

NO CALCULATORS!

1. Show using induction that the statement $P(n)$:

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{1}{3} n(n+1)(n+2)$$

is true for $n \geq 1$ showing all your steps and using compete sentences.
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2. Let

$$X = \{1, 2, 3, 4\}$$

and let

$$Y = \{5, 6, 7\}$$

a) Give an example of a function $f : X \to Y$ which is not onto. Specify f by writing out elements of $X \times Y$.

b) Explain in a complete sentence why your function is not onto.

c) What is the range of your function?

3a. Let

$$X = \{0, 1, 2, 3, 4, 5, 6\}$$

and define an equivalence relation R on X by

$$n \, R \, m \quad \text{if } n - m \text{ is divisible by } 3$$

(You need not show that this is an equivalence relation.) List all the **distinct** equivalence classes of the relation R.How many equivalence classes are there?

3b. Let

$$Y = \{0, 1, 2, 3, 4\}$$

Give an example of a relation on Y which is reflexive and transitive, but not symmetric. You should do this by listing the elements of $Y \times Y$ which make up the relation. Draw the directed graph of your relation.

4a.a) How many arrangements of the letters of INSTITUTIONAL are there?

b) How many arrangements of the letters of INSTITUTIONAL are there with the vowels in alphabetical order?

c) How many arrangements of the letters of INSTITUTIONAL are there with the vowels in alphabetical order and no consecutive N's ?

5. How many ten cards hands are there with three cards of one suit, three cards of a different suit, two cards of yet a different suit and two cards of the remaining suit? There are four suits in the deck, each with 13 cards. For example, three hearts, three spades, two diamonds, two clubs.